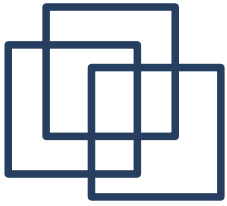


Bounding New Physics using the Tevatron Higgs Exclusion Limit

Radja Boughezal

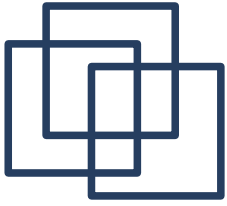
Argonne National Laboratory

Fermilab, March 3, 2011

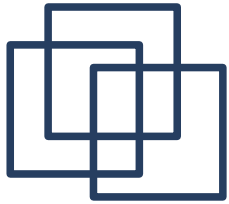


Outline

- Motivations: strength of indirect constraints
- Review of Higgs production via gluon fusion
- Looking beyond the Standard Model with the Higgs:
fourth generation of quarks, colored scalars
- Conclusions



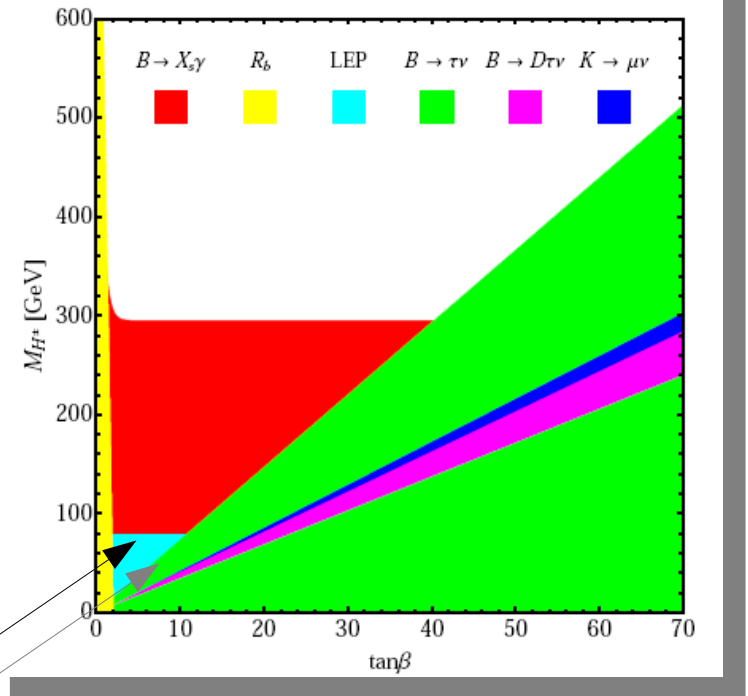
Indirect bounds on new physics can be complementary or even stronger than the direct search bounds at various colliders



Direct vs. Indirect Constraints: charged Higgs in type II THDM

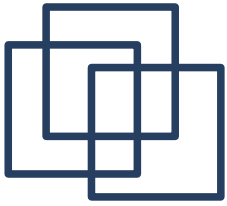
The mass of Charged Higgs boson in type II THDM has the strongest lower bound from $b \rightarrow s \gamma$ for $\tan \beta \leq 40$.

The indirect bound is stronger than the LEP direct bound.



LEP constraint

U. Haisch, arXiv:0805.2141



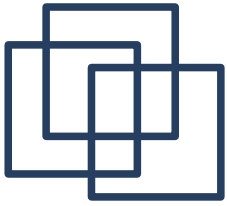
Direct vs. Indirect Constraints: Z' as an example

	EW	CDF	LEP 2
Z_χ	1,141	892	781 [21]
Z_ψ	147	878	481 [20]
Z_η	427	982	515 [21]
Z_{LR}	998	630	804 [20]
Z_{seq}	1,403	1,030	1,787 [20]

J. Erler
[arXiv:0907.0883v1](https://arxiv.org/abs/0907.0883v1)

Table 2: Lower mass limits for selected Z' bosons in GeV.

A global fit to EW precision observables provides stronger constraints on various Z' models than the direct search bounds



New physics and properties of the Higgs

New states can significantly modify the properties of the Higgs

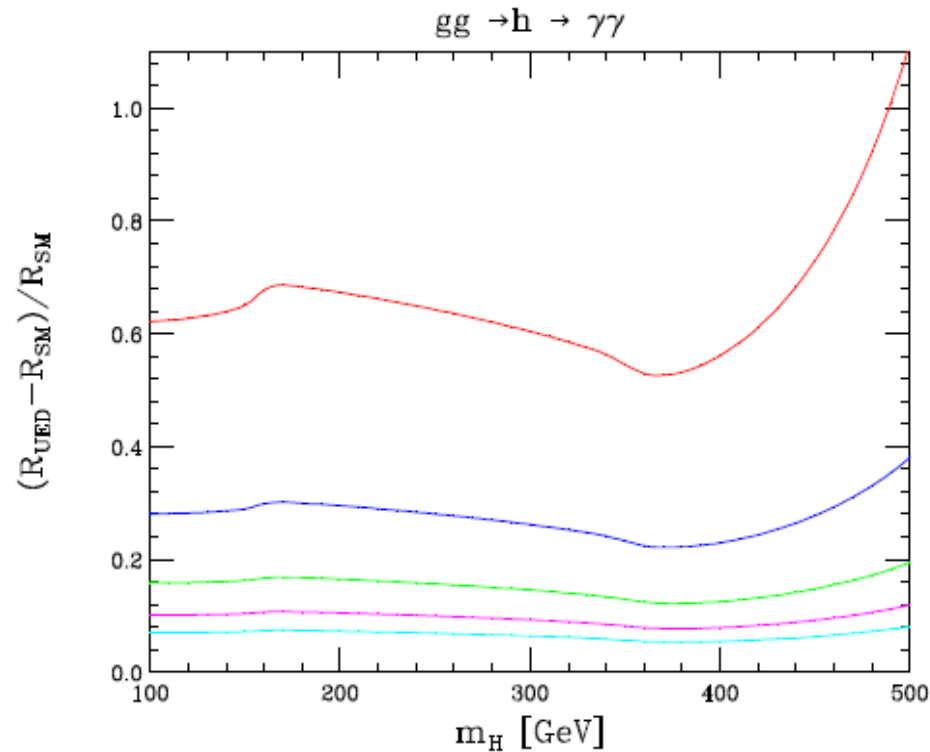
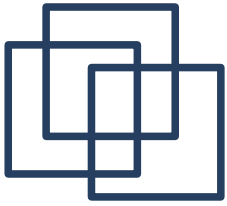


Figure 4: The fractional deviation of $R = \sigma_{gg \rightarrow h} \times \Gamma_{h \rightarrow \gamma\gamma}$, the $\gamma\gamma$ production rate, in the UED model as a function of m_H ; from top to bottom, the results are for $m_1 = 500, 750, 1000, 1250, 1500$ GeV.

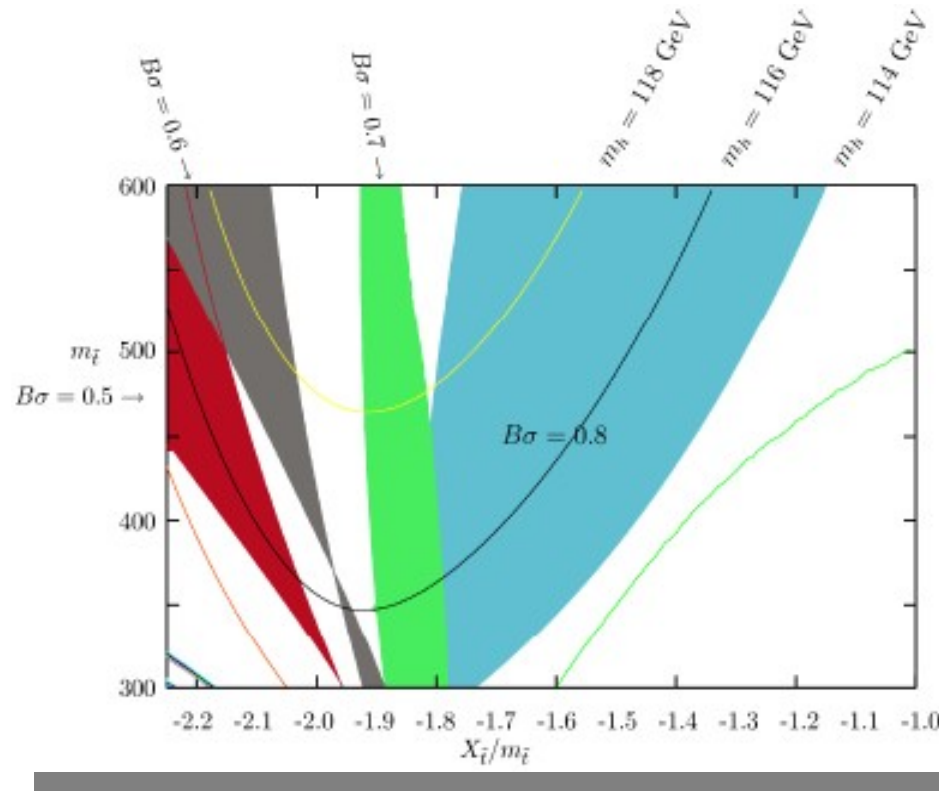


New physics and properties of the Higgs

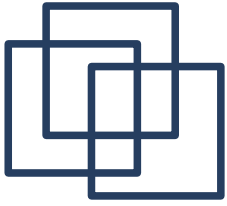
New states can significantly modify the properties of the Higgs

MSSM I. Low, S. Shalgar 2009

$$g g \rightarrow h \rightarrow \gamma \gamma$$

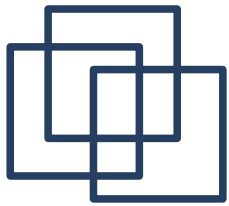


The Higgs can be very different in models beyond the SM



Can we use the Higgs boson null search results at Tevatron to indirectly learn about possible new physics ?

We need first to understand the Higgs in the SM



Current Limits on the SM Higgs Mass

Combined efforts from direct searches and theoretical predictions were needed to set tighter limits on M_H

- Current fit of electroweak parameters by LEP EW-working group predicts:

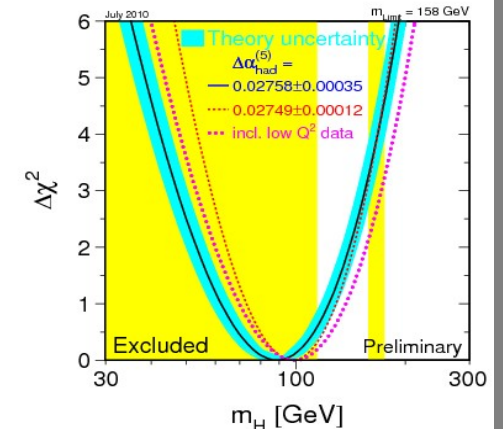
$$M_H = 89^{+35}_{-26} \text{ GeV}$$

- Upper bound (from precision EW measurements) and lower bound (direct searches at LEP) at **95% CL** (SM Higgs):

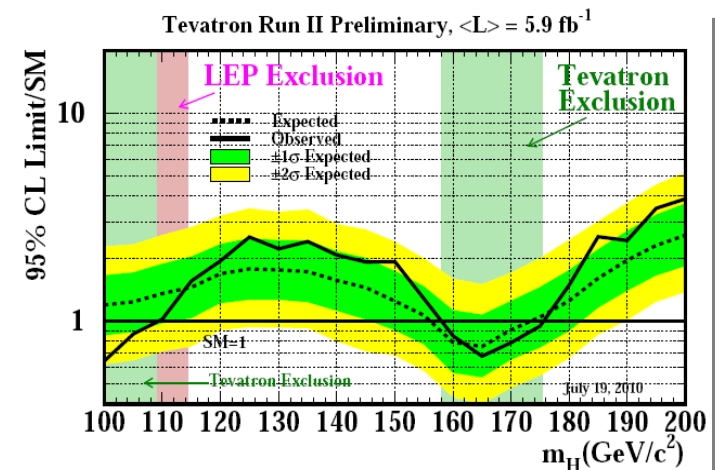
$$M_H < 158 \text{ GeV}$$

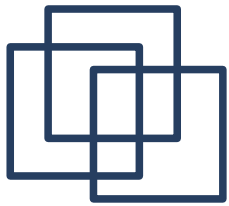
$$M_H > 114 \text{ GeV}$$

- Combined results from CDF and D0 excluded M_H in the range **158-175 GeV** and **100-109 GeV** at **95% CL** [arXiv:1007.4587](https://arxiv.org/abs/1007.4587)



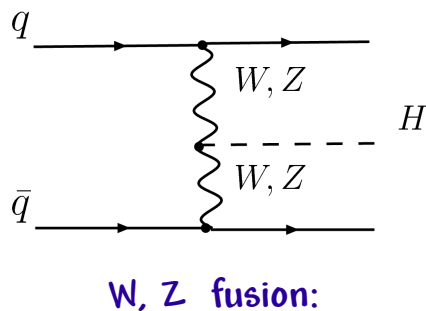
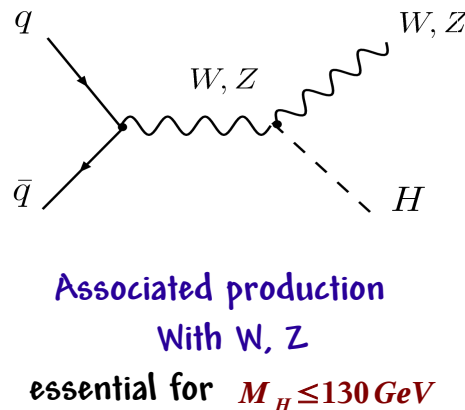
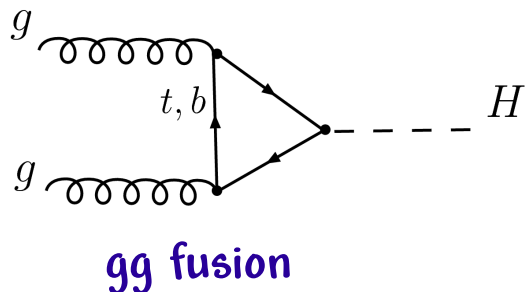
LEP EW working group July 2010



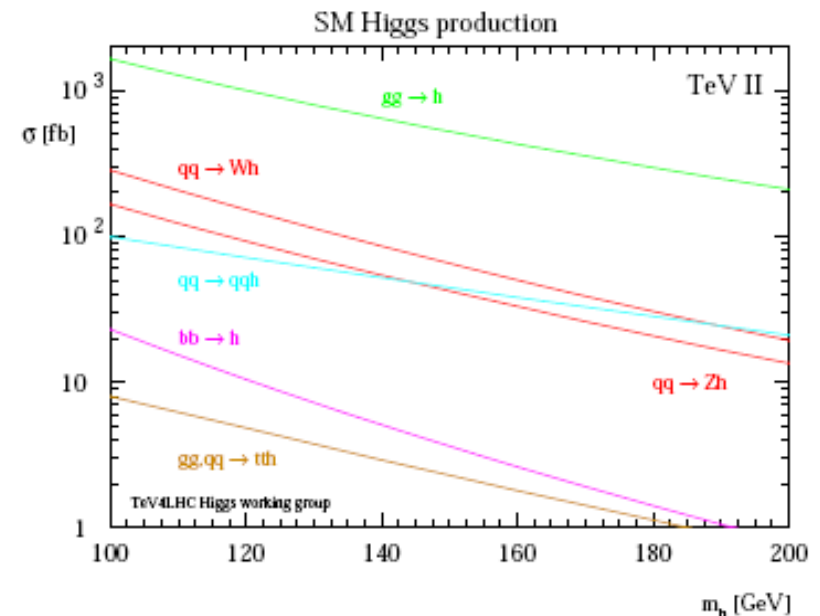
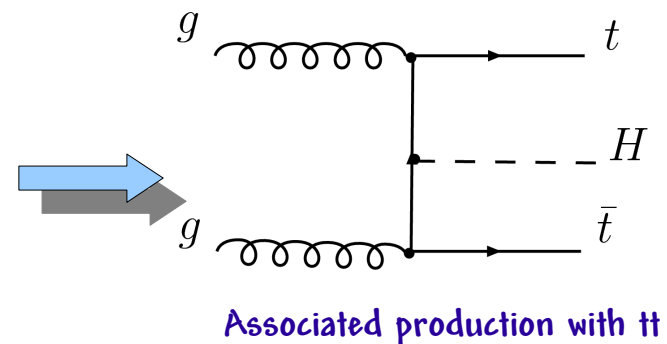


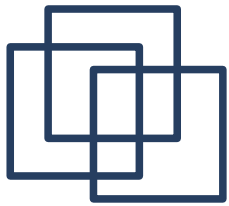
The SM Higgs Production at the Tevatron

Gluon fusion is the dominant production
Mode in the SM



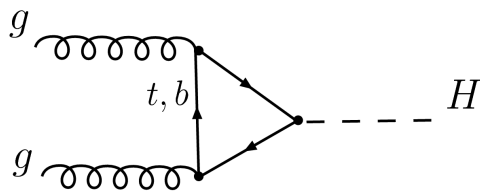
marginal process
due to its small
cross section



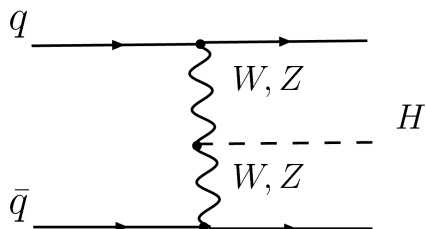
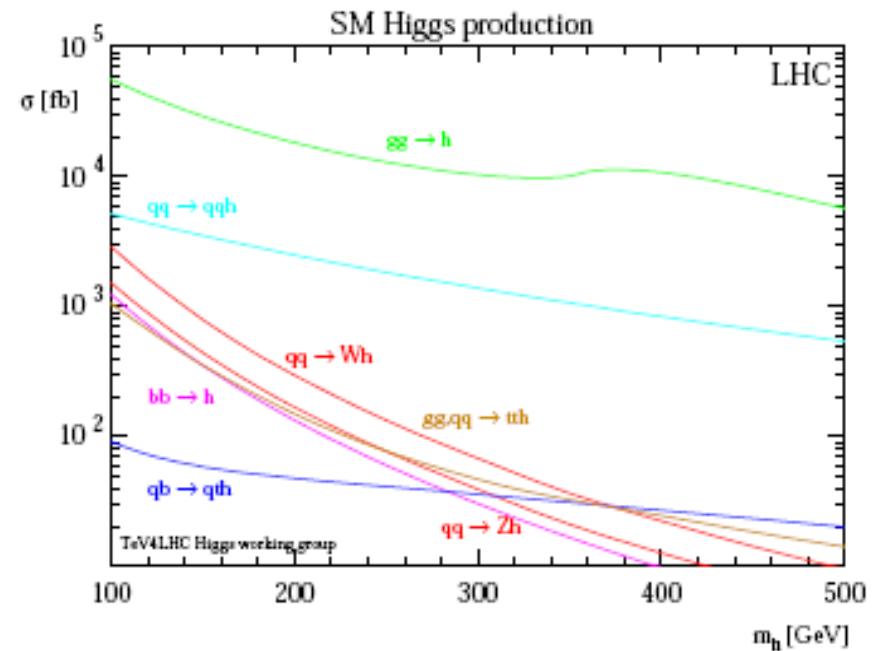


Production Mechanisms of SM Higgs at the LHC

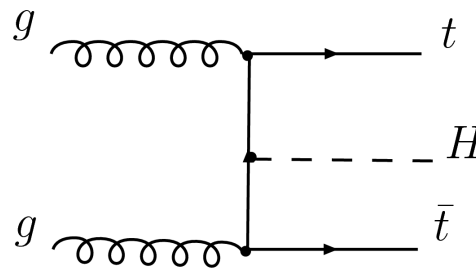
AT the LHC the SM Higgs production is also dominated by gluon fusion :



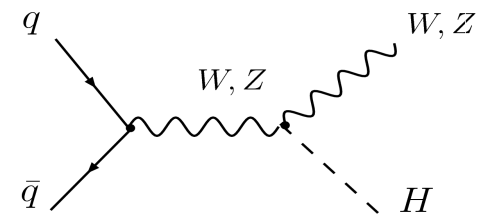
gg fusion
Dominant production mechanism
over the whole range of M_H



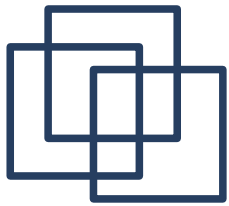
W, Z fusion:
increasingly important
at high masses



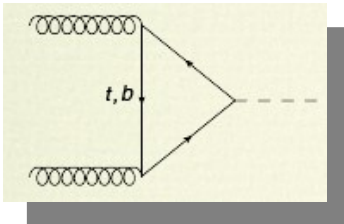
Associated production with tt
clean measurement of top-Yukawa coupling



Associated production with W, Z



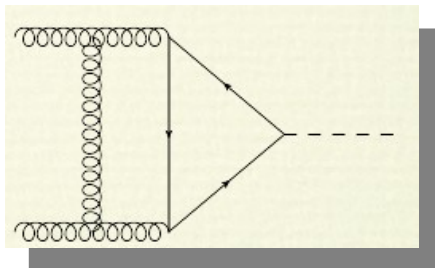
QCD Corrections to $gg \rightarrow H$



LO is one-loop \Rightarrow sensitive to new physics
BUT complicated higher order corrections

QCD @ NLO: increase LO cross section by roughly **100%**

eg. NLO graph



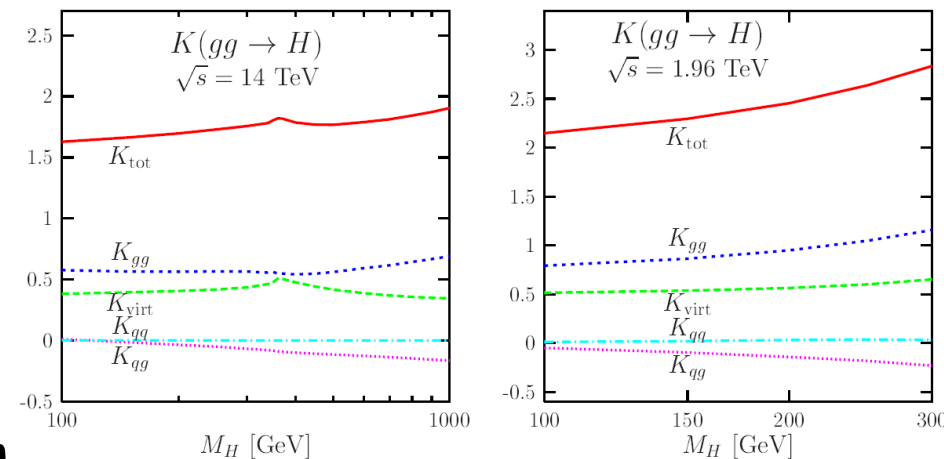
$$K = \frac{\sigma^{\text{any order}}}{\sigma^{\text{LO}}}$$

Full NLO with exact mass dependence known

Djouadi, Graudenz, Spira, Zerwas (1995);

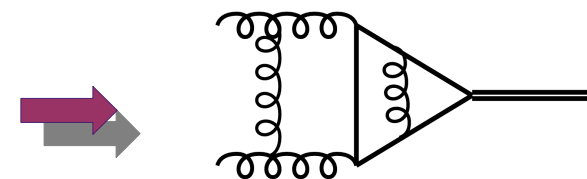
$\sigma = \sigma_0 (1 + \text{corrections})$ convergence an open question

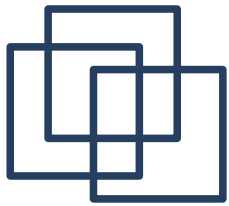
NLO K-factor



Need NNLO to check convergence of the expansion

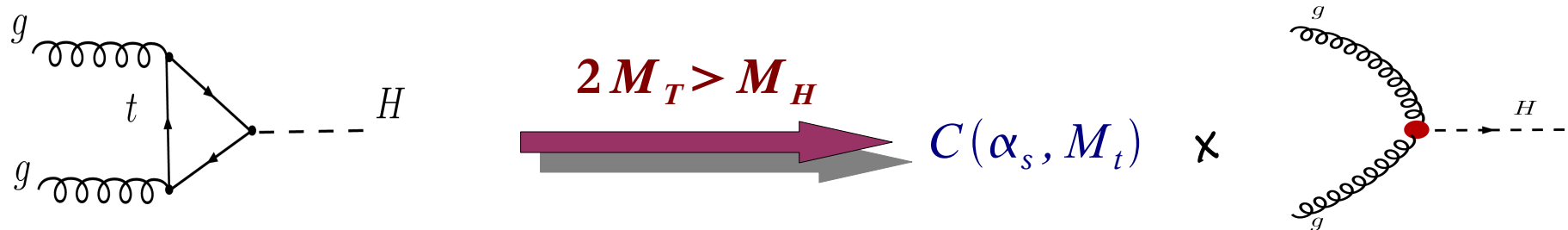
3loop vertex, 2 scales: $m_H, m_T \rightarrow$ untractable





An Effective Theory for Higgs

In the limit where the top-quark is heavier than the Higgs and all other quarks are massless, integrate out the top and couple the gluons to the Higgs through an effective vertex:



$$\mathcal{L}_{eff} = -\frac{\alpha_s}{4v} C H G_{\mu\nu}^a G^{a\mu\nu}$$

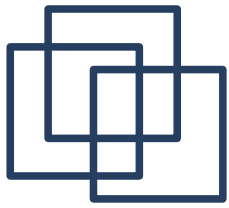
$$\left(C_0 + \left(\frac{\alpha_s}{\pi} \right) C_1 + \left(\frac{\alpha_s}{\pi} \right)^2 C_2 + \dots \right) \left(\text{gluon loop diagrams} + \dots \right)$$

Model dependent (top quark + anything)

QCD only

Factorization of QCD and model dependent effects

$C(\alpha_s)$ Known in SM through α_s^5 Schroder, Steinhauser (2006); Chetyrkin, Kuhn, Sturm (2006)



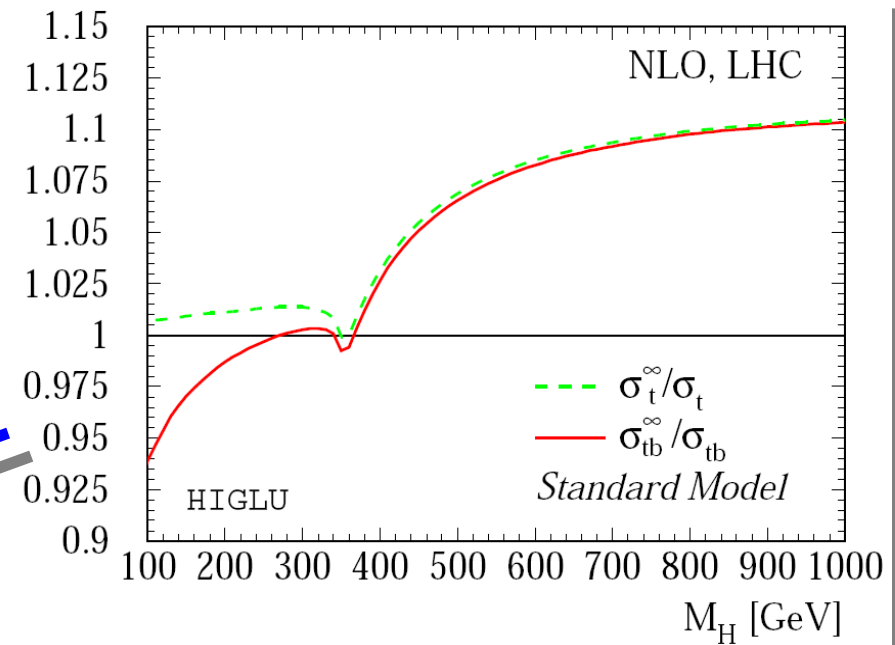
Why is the EFT approach so effective

NLO in the EFT approach: Dawson (1991); Djouadi, Spira, Zerwas (1991)

- Dominant terms to the cross section are the same in the exact and effective theory

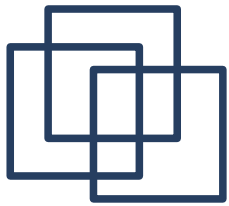
very good agreement between $\sigma^{Exact, NLO}$, $\sigma^{approximate, NLO}$
provided we normalize to the exact LO result

$$\sigma_{NLO}^{approximate} = \sigma_{QCD}^{LO}(m_t, m_b) \frac{\sigma_{NLO}^{EFT}}{\sigma_{LO}^{EFT}}$$

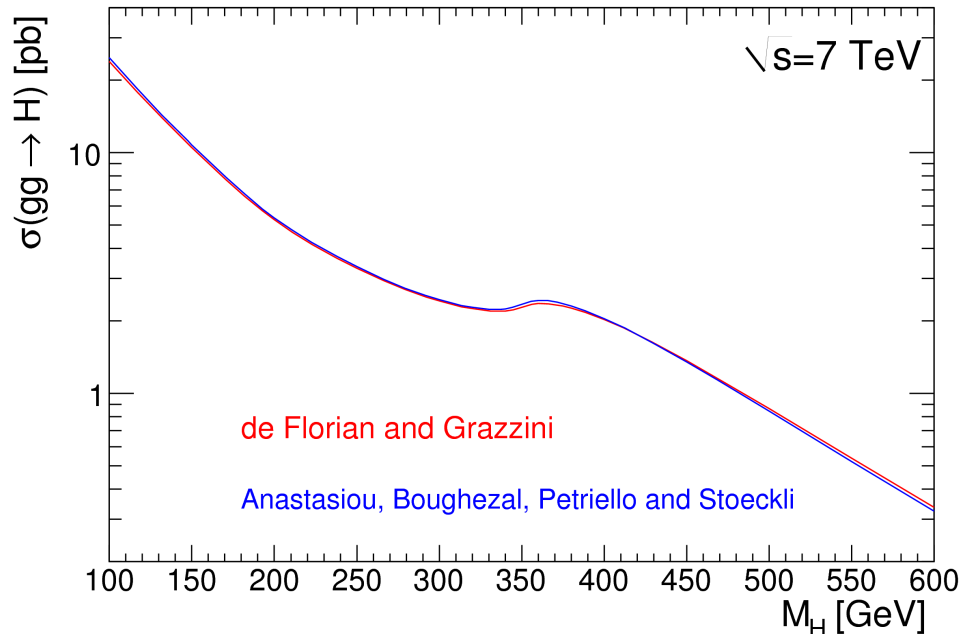


- difference $< 10\%$ for m_H up to 1 TeV
and $< 1\%$ below 200 GeV

- initial NNLO study of $1/m_t$ suppressed operators indicates this persists (Harlander et al; Pak et al, 2009)

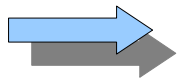


Gluon fusion predictions at the LHC

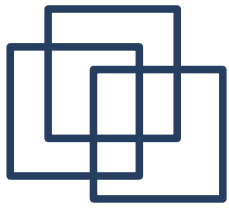


- **NNLO** QCD corrections increase xsection by **10-15%** $\sigma = \sigma_0(I + I + 0.15 + \dots)$
 - converging perturbative series
 - Reduction of renormalization and factorization scale dependence
- EW corrections increase NNLO xsection by 2-6%

Different theoretical approaches for producing Higgs predictions for $gg \rightarrow H$ were found to agree within a few percents

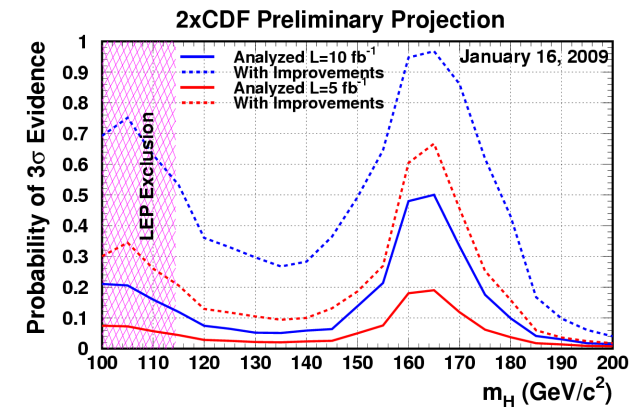
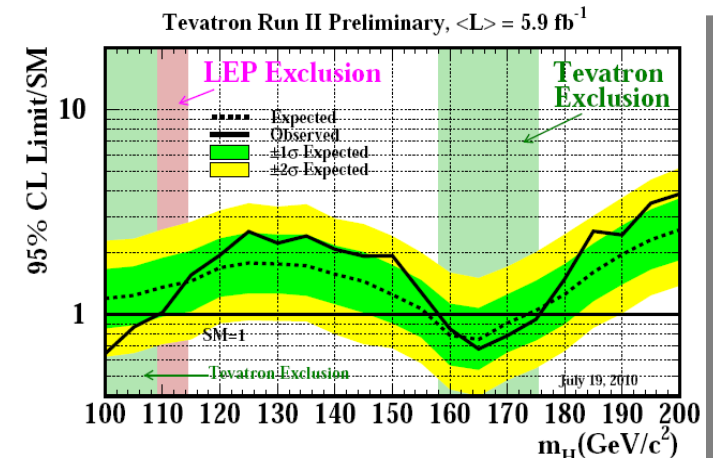


Theoretical predictions are well under control



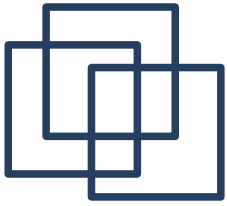
Great results from CDF and DO in both low and high mass sectors

- SM Higgs exclusion in the range 158-175 GeV @ 95% CL




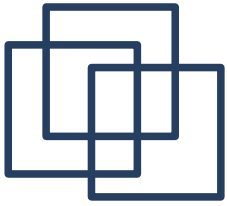
- On the theory side: theory errors have become small enough not to wash out BSM effects

Can we use these results to indirectly exclude new physics?

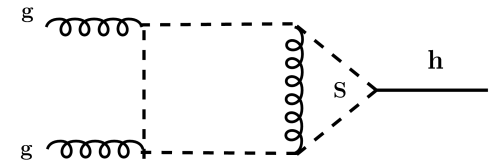
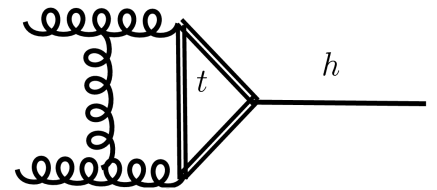
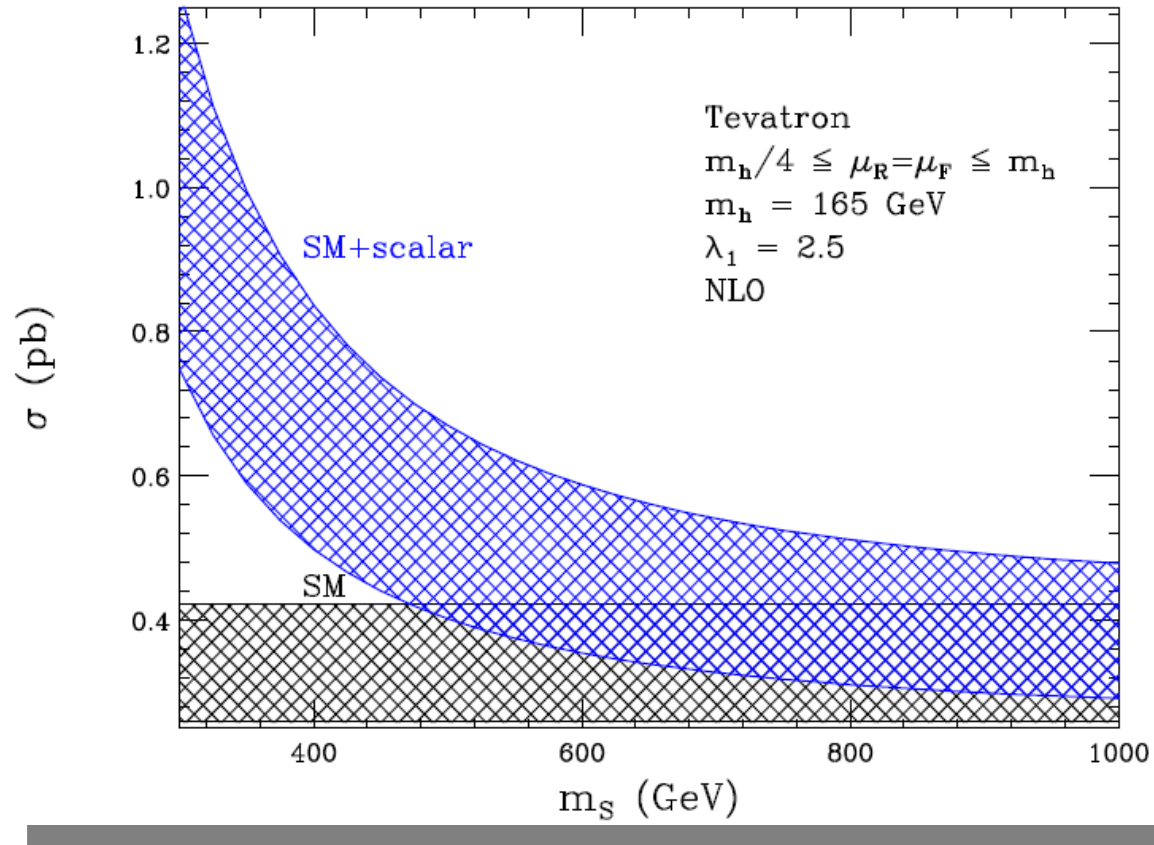


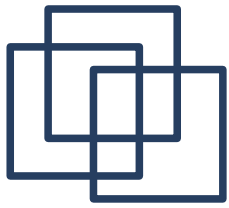
Beyond the Standard Model

- Properties of the Higgs boson can be modified in theories with additional particles
 - need precise predictions of cross sections to detect any deviations from measurements
- Higgs production via Gluon fusion is loop induced  very sensitive to new physics
- Lots of new physics to study, which Tevatron is already looking for:
 - 4th generation, colored scalar particles...
- They can couple to Higgs already at tree level and can modify the $gg \rightarrow H$ xsection

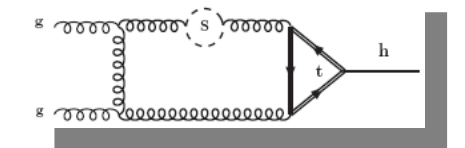
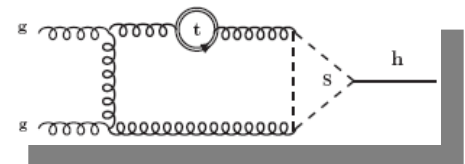
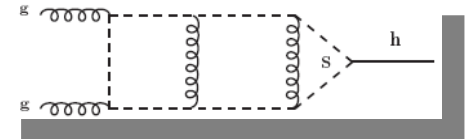
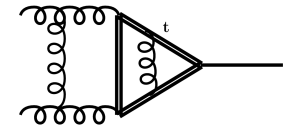
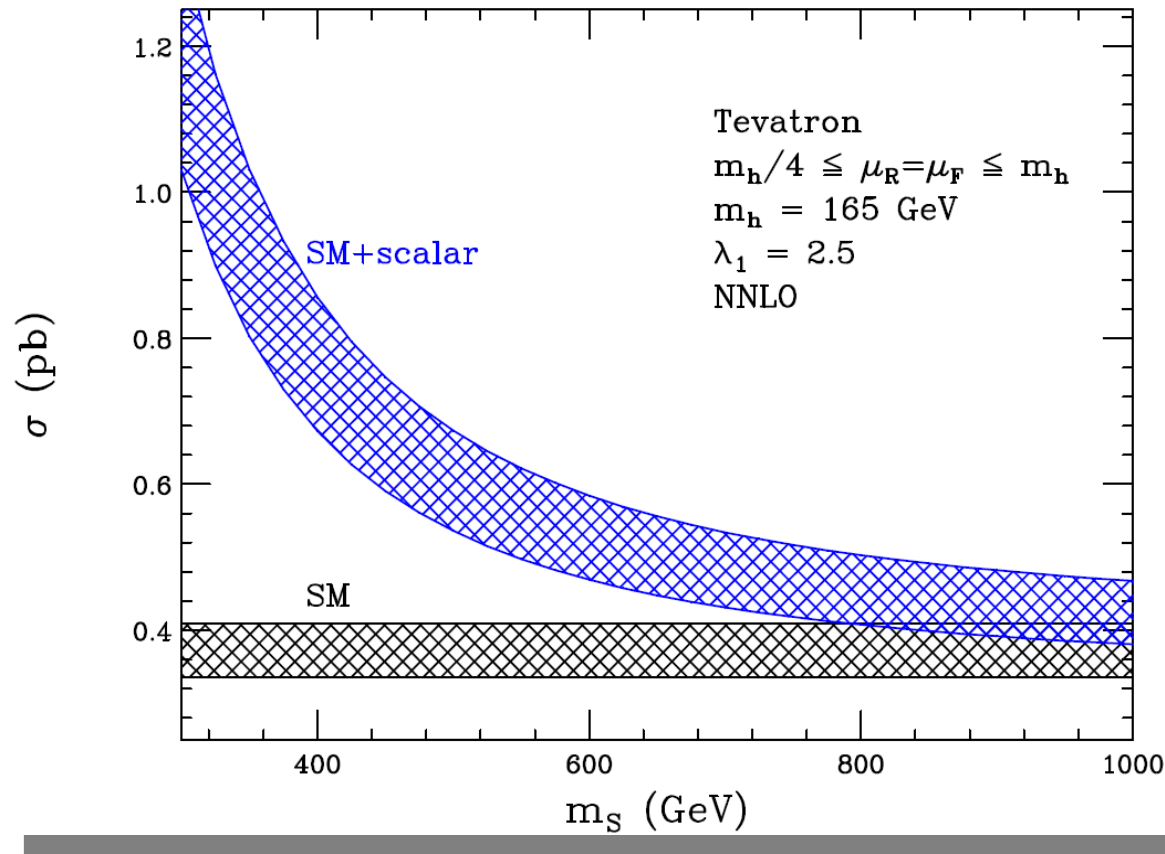


Color-adjoint scalar @ NLO

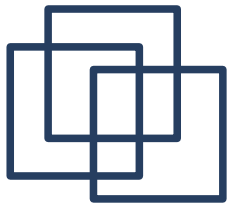




Color-adjoint scalar @ NNLO



Only at NNLO a precise prediction is obtained \rightarrow need NNLO for the indirect searches !

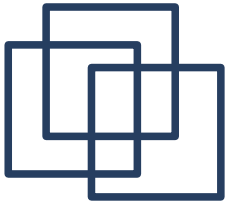


Many New Physics Possibilities

Precise predictions for lots of new physics scenarios can be provided

PARTICLES IN DIFFERENT REPRESENTATIONS OF THE LORENTZ GROUP	PARTICLES OF DIFFERENT MASS IN THE LOOPS	PARTICLES IN DIFFERENT COLOUR REPRESENTATIONS	DIFFERENT STRUCTURE OF THE HIGGS COUPLIG
QUARKS		SINGLETs, TRIPLETs, OCTETS	
SQUARKS		FUNDAMENTAL, ADJOINT	
MAJORANA FERMIONS	

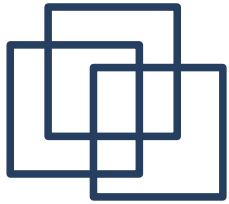
table made by E. Furlan



Example Studies:

4th generation and heavy Colored scalars effects on the cross section in the $gg \rightarrow H$ process

Details can be found in JHEP 1006:101,2010, Phys.Rev.D81:114033,2010
& arXiv:1101.3769



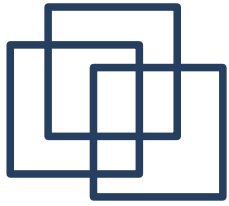
Fourth generation effects in $gg \rightarrow H$

- An experimental benchmark: fourth generation with masses larger than the SM 3 generations
 - a natural extension to the SM that can be tested with Higgs boson searches at the Tevatron
 - Precision measurements of Z boson decay width (LEP, SLD,...) excluded models with neutrino mass eigenstate **less** than **45 GeV**. A heavier fourth generation is not yet excluded

$$m_T - m_B = 50 \text{ GeV} + 10 \log \left(\frac{m_H}{115 \text{ GeV}} \right) \text{ GeV}$$

permitted by EW precision constraints
[Kribs et al arXiv:0706.3718](#)

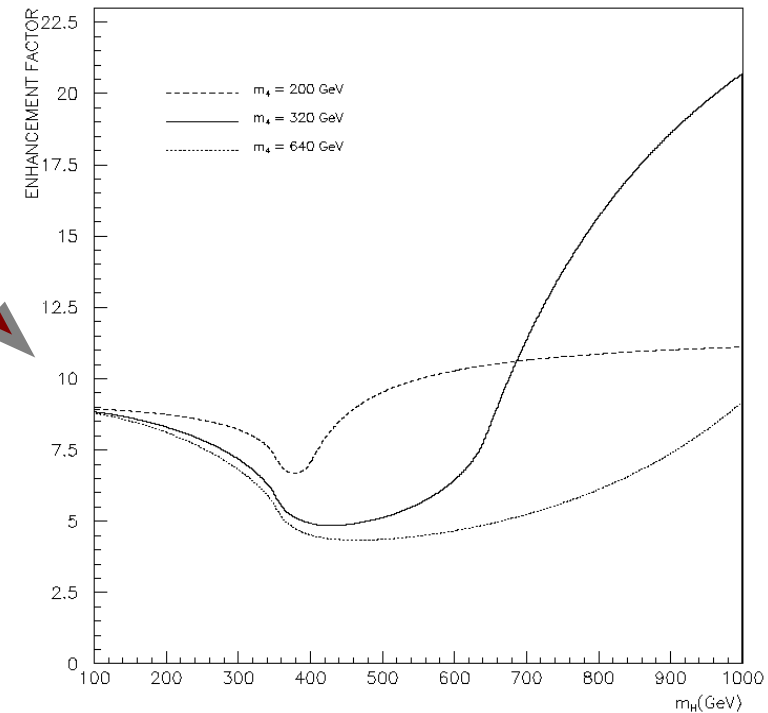
Consider QCD corrections to $gg \rightarrow H$ using a heavy doublet of quarks (T', B') in addition to the usual QCD particles



Fourth generation effects in $gg \rightarrow H$

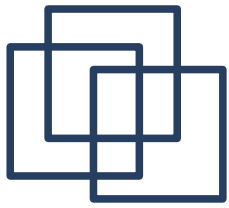
- Previous analysis was based on NLO precision:
 - infinite mass limit: $\sigma^{4,NLO} = 9 \sigma^{3,NLO}$
 - exact mass dependence (HIGLU): $\epsilon \sim 7-9$
for $100 \text{ GeV} < m_H < 300 \text{ GeV}$
- Theory uncertainty on the NLO result can change the enhancement factor and therefore the exclusion limits on the Higgs/fourth generation \rightarrow need NNLO
- Diagrams with two different heavy quarks appear for the first time at NNLO, what is their effect on the cross section?

Enhancement factor ϵ



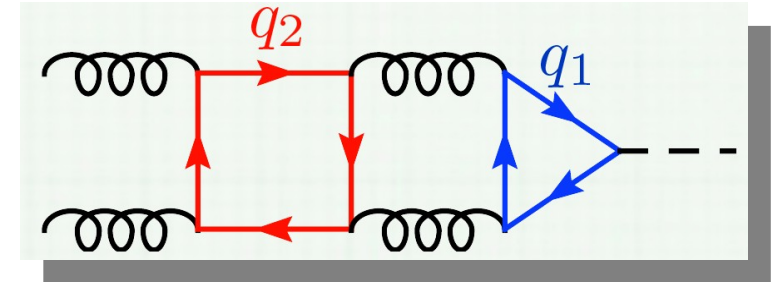
Arik, Cakir, Cetin, Sultansoy (2005)

$$\epsilon = |M_t + M_{T'} + M_B|^2 / |M_t|^2$$



Fourth generation effects in $gg \rightarrow H$

- NNLO calculation involves many loops, many scales and external legs



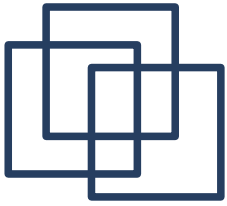
- Use an effective theory for $m_{q_1} > \frac{m_H}{2}, m_{q_2} > \frac{m_H}{2}$

$$\mathcal{L}_{eff} = -\frac{\alpha_s}{4v} \underbrace{C}_{\text{red}} \underbrace{H G_{\mu\nu}^a G^{a\mu\nu}}_{\text{blue}}$$

$$\left(C_0 + \left(\frac{\alpha_s}{\pi} \right) C_1 + \left(\frac{\alpha_s}{\pi} \right)^2 C_2 + \dots \right) \left(\text{triangle} + \text{triangle} + \dots \right)$$

$$f(m_q, m_H) = \underbrace{\left(\text{triangle} \right)}_{\text{T.E.}} = C_0 \cdot \underbrace{\left(\text{triangle} \right)}_{f(m_q)} = \underbrace{\left(\text{bubble} \right)}_{f(m_q)} \times \underbrace{\left(\text{triangle} \right)}_{f(m_q)} + \dots$$

$f(m_q, m_H)$ Taylor expand $f(m_q)$ $O(m_H^2 / 4 m_q^2)$

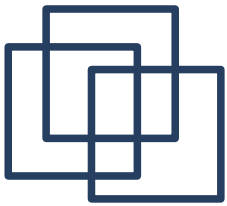


Fourth generation effects in $gg \rightarrow H$

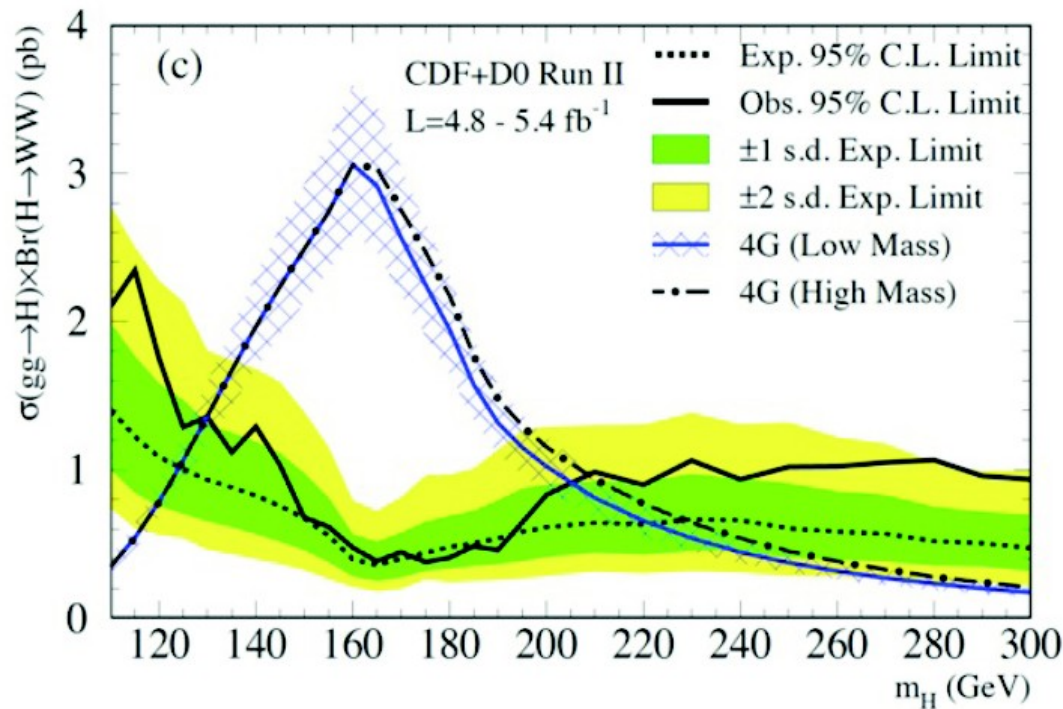
$$\frac{\sigma(gg \rightarrow H)^{(n_h)}}{\sigma(gg \rightarrow H)^{(SM)}} = \frac{\Gamma(H \rightarrow gg)^{(n_h)}}{\Gamma(H \rightarrow gg)^{(SM)}} =$$
$$n_h^2 - \left(\frac{\alpha'_s(\mu)}{\pi}\right)^2 n_h \left[\frac{77}{288} n_h (n_h - 1) + \left(\frac{4}{3} n_l + \frac{19}{4}\right) \sum_q \log \left(\frac{m_q(\mu)}{m_t(\mu)} \right) \right] + \mathcal{O}(\alpha_s'^3)$$

The contribution from NNLO  breaks simple scaling mostly due to the n_h^3 term

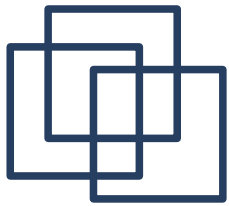
- The NNLO cross section is 10-15% higher than the NLO
- The theoretical uncertainty is decreased from 20-30% at NLO to 10% at NNLO
- This result allows the Tevatron collaboration to put accurate limits on the mass of the Higgs boson in this model



Fourth generation effects in $gg \rightarrow H$



Assuming the existence of a 4th generation of fermions with large masses,
a SM-like Higgs boson in the mass range 131-204 GeV is excluded



Model independent bounds on $\sigma(gg \rightarrow H) \times Br(H \rightarrow WW)$

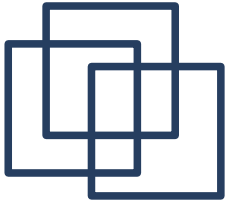
A byproduct of the 4th generation analysis of Tevatron is this interesting table:

the observed 95% CL upper limit on
 $\sigma(gg \rightarrow H) \times Br(H \rightarrow WW)$

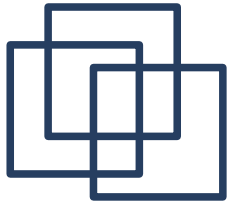
Observed limit in pb

m_H [GeV]	Obs.
110	2.10
115	2.35
120	1.75
125	1.29
130	1.36
135	1.12
140	1.29
145	1.03
150	0.68
155	0.62
160	0.47
165	0.38
170	0.45
175	0.38
180	0.41
185	0.48
190	0.46
195	0.65
200	0.83
210	0.98
220	0.90
230	1.06
240	0.93
250	1.02
260	1.02
270	1.05
280	1.07
290	0.96
300	0.93

Various new physics models can be studied using these results



*Constraints on heavy colored scalars from Tevatron's
Higgs exclusion limit*



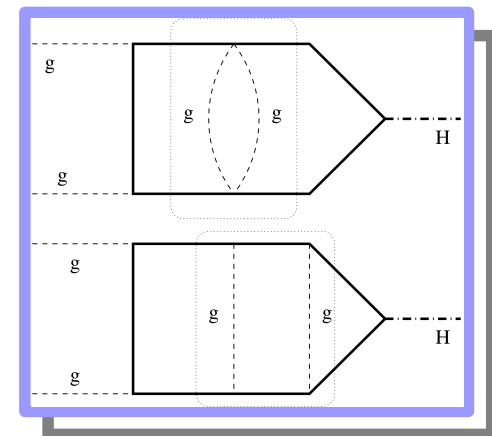
Color octet & fundamental scalars in $gg \rightarrow H$

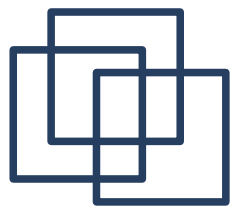
- Scalars that transform as $(8,1)_0$ and $(3,1)_0$ under $SU(3) \times SU(2) \times U(1)$

$$\begin{aligned}\mathcal{L}^{adj} &= \mathcal{L}_{SM} + \text{Tr} [D_\mu S D^\mu S] - m_S'^2 \text{Tr} [S^2] - g_s^2 G_{4S} \text{Tr} [S^2]^2 - \lambda_1 H^\dagger H \text{Tr} [S^2], \\ \mathcal{L}^{fund} &= \mathcal{L}_{SM} + (D_\mu S)^\dagger D^\mu S - m_S'^2 S^\dagger S - \frac{1}{2} g_s^2 G_{4S} (S^\dagger S)^2 - \lambda_1 H^\dagger H S^\dagger S.\end{aligned}$$

λ_1 allowed by all symmetries

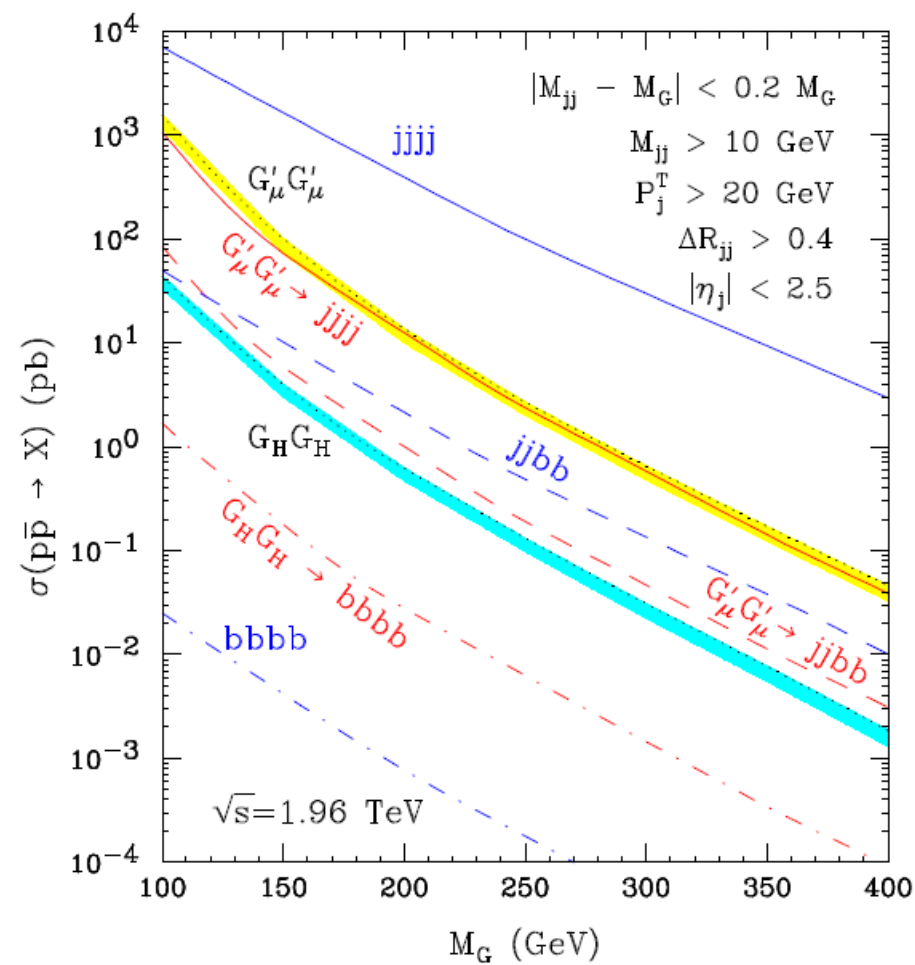
G_{4S} required by renormalizability at NNLO



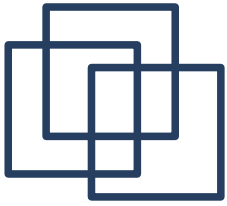


Color octet scalars in $gg \rightarrow H$

- Color octet scalars arise in theories with universal extra dimensions
- Primary decays expected to be into tt or bb depending on m_S
- Can be searched for at Tevatron by looking for four b-jet final state, BUT direct search is difficult due to large QCD background
- Search reach at Tevatron estimated to be 280 GeV (Dobrescu, Kong, Mahbubani (2007))
- Can indirectly search for it using the influence of the scalar on Higgs production xsection



Dobrescu, Kong, Mahbubani (2007)



Extracting bounds on the scalars parameter space

- Use the following LO amplitude and nth order cross section:

$$\mathcal{A}^{LO} = \mathcal{A}_t^{LO} + \mathcal{A}_b^{LO} + \mathcal{A}_S^{LO}$$

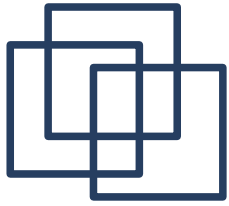
$$\sigma^n = \sigma_{t+S}^{LO}(m_t, m_S) K_{EFT}^n + \sigma_{Sb}^{LO}(m_S, m_b) + \sigma_{tb}^{LO}(m_t, m_b) + \sigma_{bb}^{LO}(m_b)$$

- Use HDECAY to produce the SM partial decay widths of the Higgs

$$\Gamma_{gg}, \Gamma_{\gamma\gamma}, \Gamma_{Z\gamma}, \Gamma_{WW}, \Gamma_{ZZ}, \dots$$

- Replace Γ_{gg}^{SM} with the one that includes the scalar contribution Γ_{gg}^{new}
- The scalars increase the Higgs production cross section and the gg partial width

How does this change the $BR(H \rightarrow WW)$?



Extracting bounds on the scalars parameter space

Example: $\Gamma_{g g}^{new} = 5 \Gamma_{g g}^{SM}$

$M_H = 120 \text{ GeV}$

$$Br(H \rightarrow WW)^{SM} = 0.13$$

$$Br(H \rightarrow WW)^{new} = 0.099$$

Roughly 25% decrease

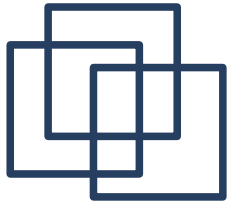
$M_H = 165 \text{ GeV}$

$$Br(H \rightarrow WW)^{SM} = 0.9581$$

$$Br(H \rightarrow WW)^{new} = 0.946$$

Roughly 1% decrease

The branching ratio is mostly affected at low Higgs masses where it decreases significantly



Extracting bounds on the scalars parameter space

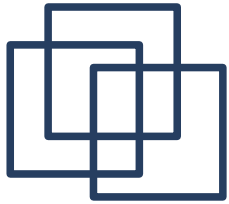
Two competing effects:

- an increasing cross section for all values of m_H
- a branching ratio that decreases at low m_H and remains almost unchanged at high m_H

Implications:

- the stronger bounds are obtained at higher values of m_H
- bounds at low values of m_S (< 50 GeV) should not be taken seriously due to the limitation of the effective theory

Note: included a constraint $\frac{\Gamma_{tot}}{m_H} < \frac{1}{5}$ to prevent strong couplings

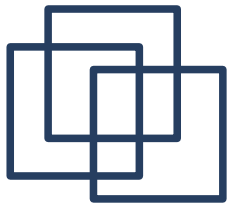


Extracting bounds on the scalars parameter space

- The scalar sector is defined through the parameters λ_1, G_{4S}, m_S
- Use RGE to get the allowed values of G_{4S} by demanding absence of Landau pole up to 10 TeV:
 - adjoint scalar $G_{4S}(v) < 1.5$
 - fundamental scalar $G_{4S}(v) < 2.5$

we chose $G_{4S} = 1$ and checked that other values in the allowed range change the bounds by at most 5%

- There is no symmetry reason to expect λ_1 to be small.
we chose $\lambda_1 = 1$ for simplicity

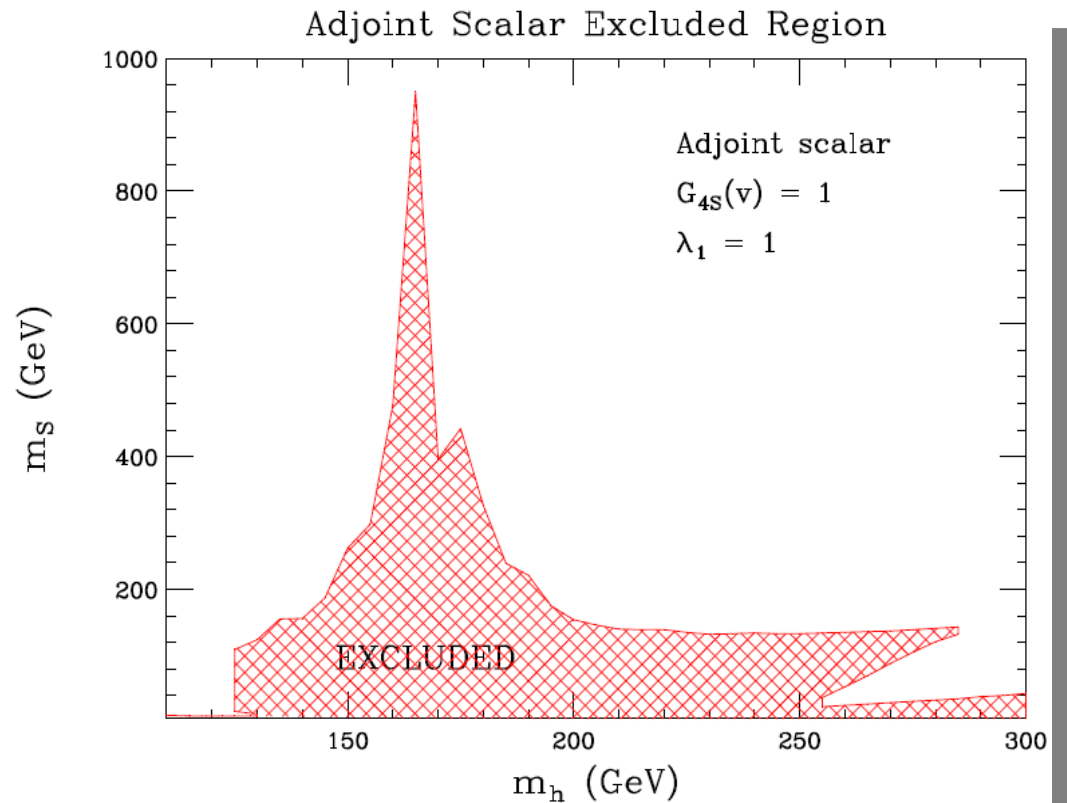


Extracting bounds on the scalars parameter space

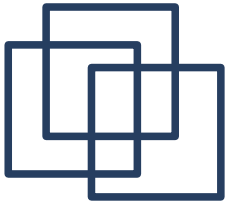
- Strongest bound occurs at $m_H = 165 \text{ GeV}$

$$m_S^{adj} \geq 900 \text{ GeV}$$

- Excluded $m_S < 130 \text{ GeV}$ for $135 < m_H < 250 \text{ GeV}$
- Estimated direct search limit is 280 GeV at Tevatron for scalars decaying primarily to $b\bar{b}$



Direct search insensitive to m_H and λ but depends on the decay mode while indirect search is independent from the decay mode but sensitive to m_H and λ

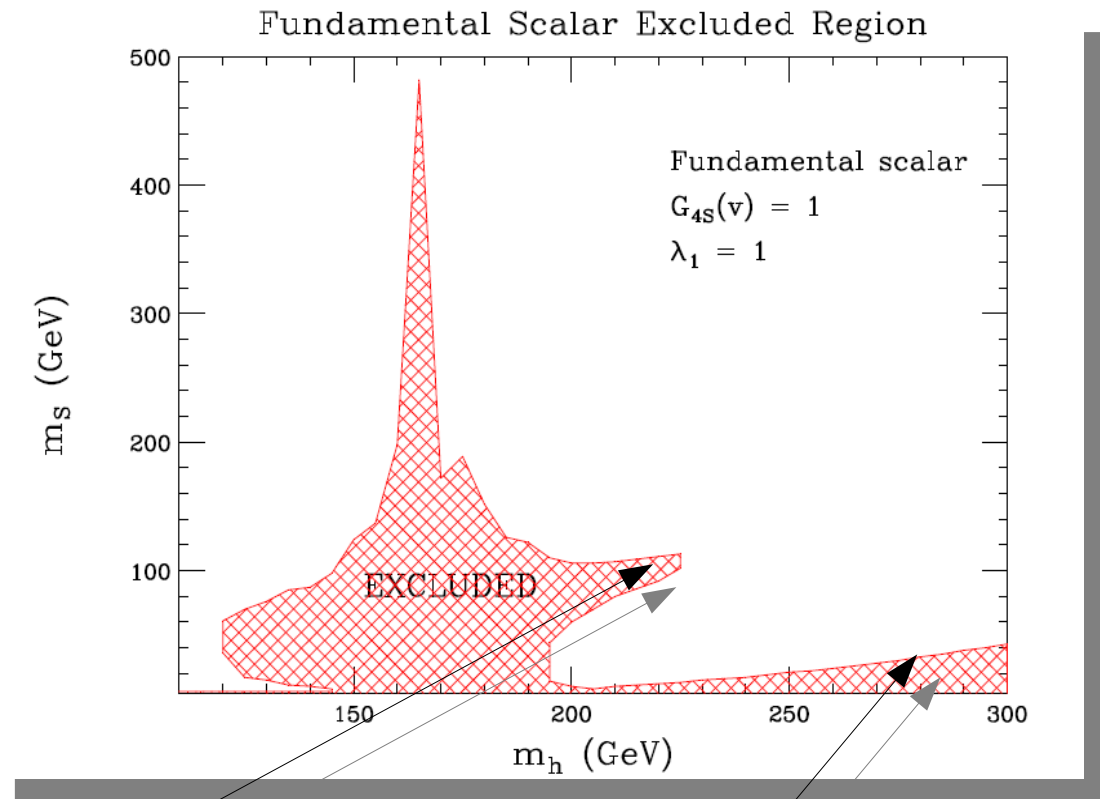


Extracting bounds on the scalars parameter space

- Strongest bound occurs at $m_H = 165 \text{ GeV}$

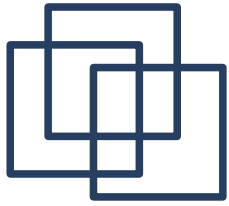
$$m_S^{\text{fun}} \geq 500 \text{ GeV}$$

- Excluded $m_S < 100 \text{ GeV}$ for $150 < m_H < 190 \text{ GeV}$



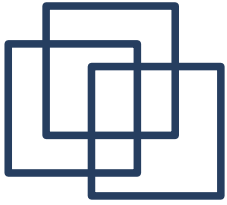
Threshold enhancement for
 χ section for $m_H = 2 m_S$

Tail comes from
 constraint $\frac{\Gamma_{\text{tot}}}{m_H} < \frac{1}{5}$

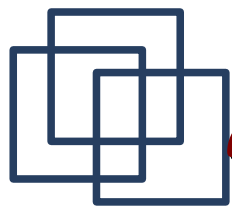


Summary

- Direct and indirect search techniques are complementary for probing new physics parameter space
- The precision of the $gg \rightarrow H$ prediction in SM reached the level where new physics effects can not be washed out. This has become an additional constraint on physics Beyond the SM
- I have showed two example states that significantly alter the Higgs cross section: color-adjoint and color-fundamental states
 - strong constraints on their parameter space were obtained using Tevatron's exclusion limit for $gg \rightarrow H \rightarrow WW$
 - many other models involving heavy colored particles coupled to Higgs can be studied and constrained in a similar way



Backup Slides

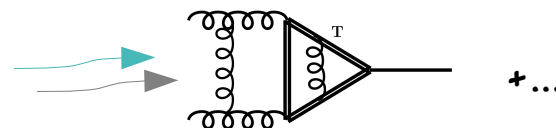


Color Octet scalar effects in $gg \rightarrow H$: the Wilson coefficient

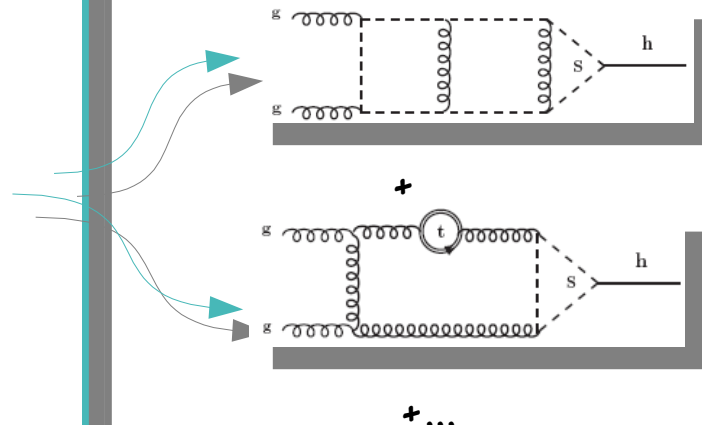
The NNLO Wilson coefficient for the adjoint scalar

$$C_1 = C_{TTH} + C_{SSH} + C_{TS}$$

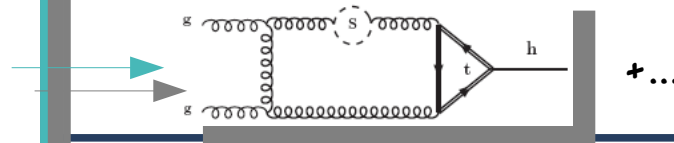
$$C_{TTH} = -\frac{a'}{3} - \frac{11 a'^2}{12} + a'^3 \left[\frac{1}{864} (-2777 + 684 L_T) + \frac{1}{288} (67 + 64 L_T) n_l \right]$$

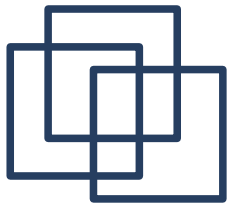


$$\begin{aligned} C_{SSH} = & -\frac{\lambda_1 v^2}{2 m_S^2} \left\{ \frac{a'}{4} + a'^2 \left[\frac{33}{16} + \frac{5 G_{4S}}{8} \right] + a'^3 \left[n_l \left(\frac{-101}{288} + \frac{7 L_S}{24} \right) \right. \right. \\ & + G_{4S}^2 \left(\frac{-35}{16} + 5 L_S \right) + \frac{9 L_S (-43 + 8 x^2)}{64} - \frac{3 (76 - 3895 x^2 + 257 x^4)}{1024 x^2} \\ & - G_{4S} \left(\frac{-705}{64} + \frac{575 L_S}{96} + \frac{5 \ln(x)}{24} \right) + \frac{3 (76 + 37 x^2 + 86 x^4 + 225 x^6)}{2048 x^3} \times \\ & \left. \left(\text{Li}_3(x) - \text{Li}_3(-x) \right) \right. \\ & + \ln^2(x) \left\{ -\frac{228 + 41 x^2 - 192 x^4 + 675 x^6}{2048 (-1+x) x^2 (1+x)} + \frac{3 (76 + 37 x^2 + 86 x^4 + 225 x^6)}{4096 x^3} \times \right. \\ & \left. \left. \left(\ln(1+x) - \ln(1-x) \right) \right\} \right. \\ & \left. + 3 \ln(x) \left\{ \frac{76 - 111 x^2 + 159 x^4}{1024 x^2} - \frac{76 + 37 x^2 + 86 x^4 + 225 x^6}{2048 x^3} \left(\text{Li}_2(x) - \text{Li}_2(-x) \right) \right\} \right\} \end{aligned} \quad (3.22)$$



$$\begin{aligned} C_{TS} = & a'^3 \left[\frac{9 L_S x^2}{8} - \frac{2052 + 1075 x^2 + 1755 x^4}{9216 x^2} \right. \\ & + \ln(x) \left\{ \frac{684 + 409 x^2 + 1431 x^4}{3072 x^2} - \frac{3 (76 + 37 x^2 + 86 x^4 + 225 x^6)}{2048 x^3} \left(\text{Li}_2(x) - \text{Li}_2(-x) \right) \right\} \\ & + \ln^2(x) \left\{ -\frac{228 + 41 x^2 - 192 x^4 + 675 x^6}{2048 (-1+x) x^2 (1+x)} + \frac{3 (76 + 37 x^2 + 86 x^4 + 225 x^6)}{4096 x^3} \times \right. \\ & \left. \left. \left(\ln(1+x) - \ln(1-x) \right) \right\} + \frac{3 (76 + 37 x^2 + 86 x^4 + 225 x^6)}{2048 x^3} \left(\text{Li}_3(x) - \text{Li}_3(-x) \right) \right] \end{aligned} \quad (3.23)$$





Method

Expansion by subgraphs (Chetykin; Gorishny; V. A. Smirnov)

$$\begin{aligned}
 & \text{Triangle Diagram} = \text{T.E.}_1 + \text{T.E.}_2 + \text{T.E.}_3 \\
 & = \left(\frac{\alpha_s}{\pi}\right)^2 C_2 \cdot \text{Gluon Tadpole} + \frac{\alpha_s}{\pi} C_1 \cdot \text{Gluon Tadpole} + C_0 \cdot \text{Ghost Tadpole}
 \end{aligned}$$

- Expand in all the momenta external to $F = \text{any subgraph}$
- Expand in the external momenta p_1, p_2
- All the reduced graphs (no heavy scale dependence) are known from SM calculations

$$\mathcal{F} = \sum_{n=0}^{\infty} \mathcal{F}_n (p_1 \cdot p_2)^n, \quad \mathcal{F}_n = \mathcal{D}_n \mathcal{F} \Big|_{p_1=p_2=0} \left(\mathcal{D}_0 = 1, \mathcal{D}_1 = \frac{1}{d} \square_{12}, \dots \right)$$